Suppose events form a partition of the sample space . That is, they are disjoint and

Assume that . Prove that if then for some .

*Proof.*  Suppose to the contrary that and for each . This means,

and

for each . Given , then (for each )

Consequently, seeing that , we must have

We know that each is disjoint, so whenever . Simultaneously, we know that for each . As such whenever . That is to say, each is disjoint. By the third axiom of probability†,

Recall that the collection of each forms a partition of , so . Furthermore, by the distributive property of set intersection over set union, we have . Ultimately,

a contradiction. It must then be the case that at for some .

† - We assume that -additivity implies finite additivity.